CHEKLI GRUPPANING GRAFI VA UNING XOSSALARI.

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Bizga bitta binar T va bitta unar $*$ algebraik amallarga ega bo'lgan bo'sh bo'lmagan G to'plam berilgan bo'lsin.

1-ta'rif. Agar G to'plamda quyidagi

1) $\forall\, a, b, c \in G,\ \ a\,T\,(b\,T\,c) = (a\,T\,b)\,T\,c$, ya'ni T binar algebraik amal assotsiativ;

2) $\forall\, a \in G,\ \ \exists\, e \in G,\ \ a \perp e = a = eTa$, ya'ni T algebraik amalga ko'ra har bir $a \in G$ element uchun o'ng va chap e neytral element mavjud;

3) $\forall\, a \in G,\ \exists\, a^* \in G, a\,T\,a^* = e = a^*\,T\,a$, ya'ni istalgan $a \in G$ uchun o'ng va chap simmetrik element mavjud [1],

aksiomalar bajarilsa, u holda $< G, T, * >$ algebra gruppa deyiladi.

1-misol. Barcha butun sonlar to'plami Z, "+" amaliga nisbatan gruppa bo'lishini tekshiramiz. Z ning elementlari uchun qo'shish amali aniqlanganligi sababli bu to'plam additiv gruppaning barcha aksiomalarini bajaradi [2-3].

1. Assotsiativlik: $\forall\, a, b, c \in Z,\ \ \ a + (b + c) = (a + b) + c;$
2. Neytral element: $\exists\, e = 0 \in Z$ shundayki $a + 0 = a = 0 + a;$
3. Simmetrik element: $\forall\, a \in Z,\ \exists - a \in Z,\ a + (-a) = 0 = (-a) + a.$

Demak, $< Z, + >$ gruppa ekan.

2-ta'rif. Agar G gruppaning elementlari soni chekli bo'lsa, u holda G gruppa chekli gruppa deyiladi. Chekli gruppaning elementlari soni uning tartibi deyiladi va $|\,G|$ kabi belgilanadi.

Elementlari cheksiz ko'p bo'lgan gruppalar cheksiz gruppalar deyiladi.

3-ta'rif. G gruppaning $a \in G$ elementi uchun $a^n = e$ shartni qanoatlantiruvchi natural sonlarning eng kichigiga berilgan elementning tartibi deb ataladi. Agar $a^n = e$ shartni qanoatlantiruvchi natural son mavjud bo'lmasa, u holda bu elementning tartibi cheksizga teng deb olinadi. Berilgan $a \in G$ elementning tartibi $ord(a)$ kabi belgilanadi.

Aytaylik $< G,\ * >$ – biror chekli gruppa bo'lsin.

$\forall\, a,\ b\ \in G$ uchun $a * b \in G$;

$\forall\, a,\ b,\ c\ \in G$ uchun $a * (b * c) = (a * b) * c$;

$\exists\, e \in G{:}\,(yagona)\ \forall a \in G$ uchun $e * a = a * e = a$;

$\forall a \in G$ uchun $\exists a^{-1} \in G{:}\,(yagona)\ a * a^{-1} = a^{-1} * a = e.$

Agar $< G, \ast >$ gruppaning elementlari soni $|G| = n$ ga teng bo‘lsa, u holda $< G, \ast >$ gruppaning ixtiyoriy ikkita elementlari orasidagi bog‘lanishni ($\ast$ – amalni) hisoblash mumkin.

$< G, \ast >= \{e, a, a^2, \dots, a^{n-1}\}$, e-birlik element va $a^n = e$ ko’paytirish amaliga nisbatan

$$e \cdot e = e$$
$$a \cdot e = a$$
$$a \cdot a = a^2$$
$$a \cdot a^2 = a^3$$

…

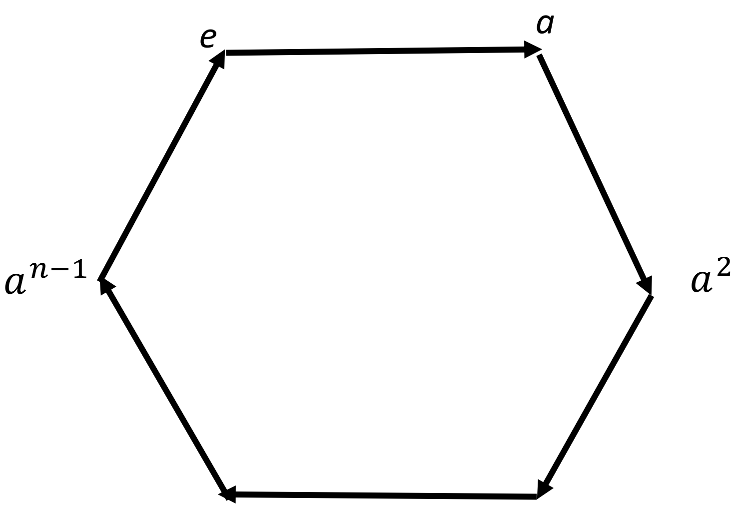
$$a^{n-1} \cdot a^{n-1} = a^{n-2}$$

Ushbu ko’paytirishni quyidagi jadvalda to’ldiramiz (1-jadval).

$\cdot$	e	a	a^2	…	a^{n-1}
e	e	a	a^2	…	a^{n-1}
a	a	a^2	a^3	…	
a^2	a^2	a^3	a^4	…	a
$\vdots$	$\vdots$	$\vdots$	$\vdots$	… … … …	$\vdots$
a^{n-1}	a^{n-1}	e	a	…	a^{n-2}

1-jadval

$< G, \ast >= \{e, a, a^2, \dots, a^{n-1}\}$ siklik gruppaning grafi muntazam n burchak

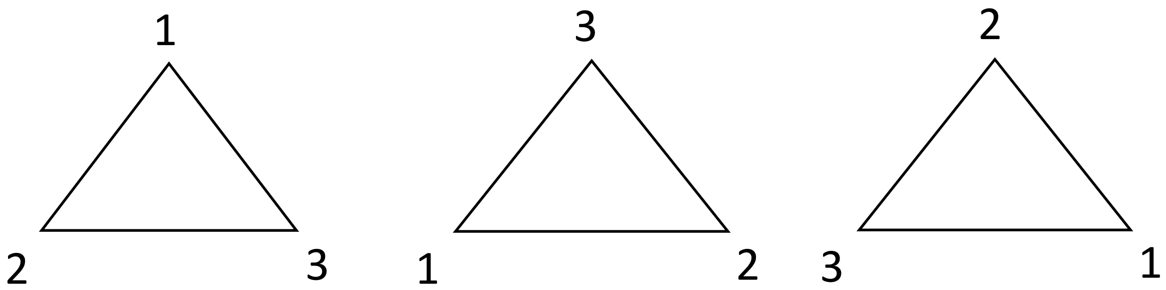


bo‘ladi [4-6].

1-chizma

Aytaylik tekislikda muntazam uchburchak berilgan va uning uchlari 1, 2, 3 natural sonlar bilan belgilangan bo‘lsin. Ushbu uchburchakni soat stirelkasiga qarshi $0^{\circ}, 120^{\circ}, 240^{\circ}$ graduslarga burilsa, 3 xil uchburchak hosil bo‘ladi (2-chizma). Uchburchak 360° ga burilganda uchburchak dastlabki holatiga qaytadi. Demak, uchburchak 120° ga burishga nisbatan siklik gruppani tashkil etar ekan.

$$0^{\circ} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \qquad 120^{\circ} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \qquad 240^{\circ} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

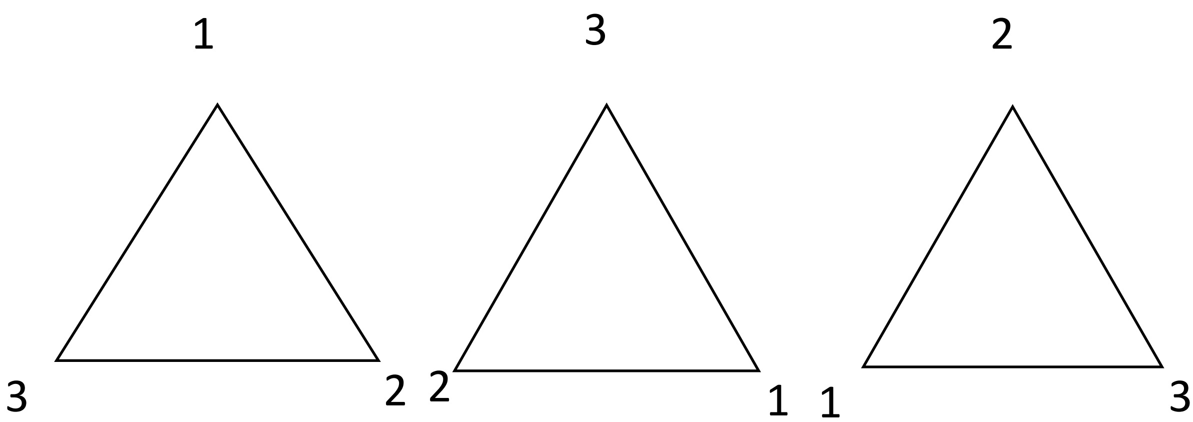


2-chizma

Endi uchburchakni mediana (balandlik) siga nisbatan simmetrik almashtirsak, yuqoridagi 3 ta element quyidagi elementlarga almashadi (3-chizma).

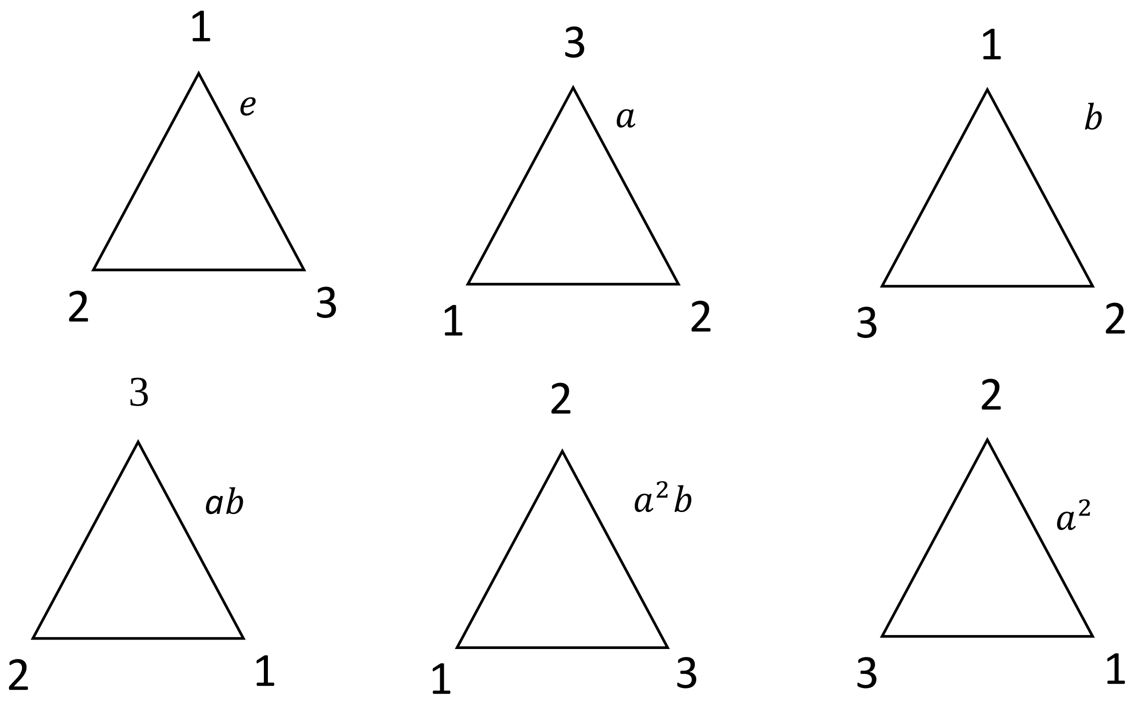
$$(2,3) = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \qquad (1,3) = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, \qquad (1,2) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}.$$

3-chizma



Bunda simmetrik almashtirish tartibi ikkiga teng bo‘ladi.

Agar a – uchburchakni soat stirelkasiga qarshi 120° ga burish, b – uchburchakni simmetrik almashtirsak deb olsak, u holda yuqoridagi 6 ta elamentni a va b elementlar yordamida quyidagicha belgilashimiz mumkin (4-chizma).



4-chizma

$e = 0^{\circ} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$, $a = 120^{\circ} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$, $a^2 = 240^{\circ} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

$b = (2{,}3) = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$, $ab = (1{,}3) = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$, $a^2b = (1{,}2) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$.

Bundan ko'rinadiki, $ab \neq ba$, ya'ni Abel (kommutativ) gruppa emas.

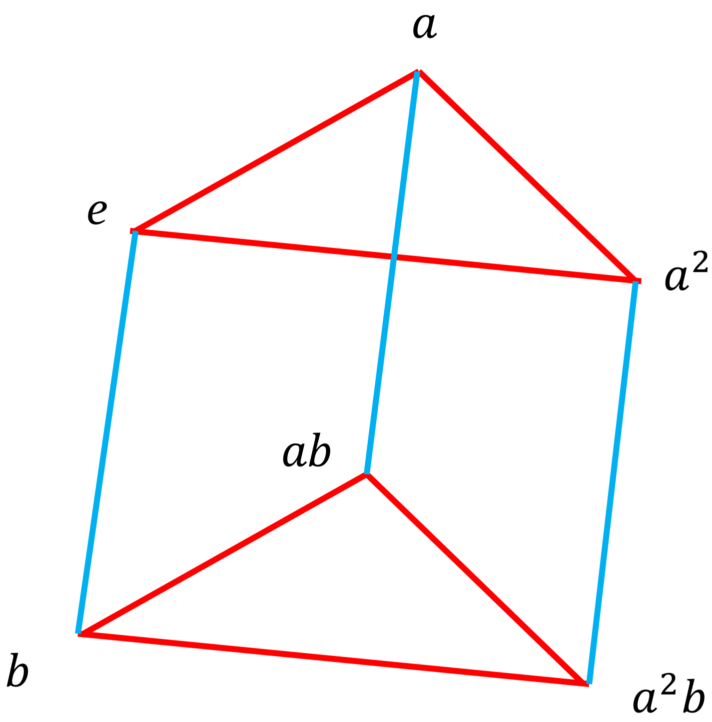
Endi bu gruppaning 6 ta elementlari orasidagi bog'lanishdan quyidagi jadvalni tuzishimiz mumkin (2-jadval).

$\cdot$	e	a	a^2	b	ab	a^2b
e	e	a	a^2	b	ab	a^2b
a	a	a^2	e	ab	a^2b	b
a^2	a^2	e	a	a^2b	b	ab
b	b	a^2b	ab	e	a^2	a
ab	ab	b	a^2b	a	e	a^2
a^2b	a^2b	ab	b	a^2	a	e

2-jadval

Endi 6 ta elemenli gruppaning grafini tasvirlaymiz. Bu grafni a va b yasovchi elementlar orqali hosil qilamiz (5-chizma).

5-chizma



Foydalanilgan adabiyotlar

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