CHIZIQSIZ KUCHSIZ MAXSUSLIKKA EGA YADROLI IKKINCHI TUR VOLTER TIPIDAGI INTEGRAL TENGLAMALARNI ODDIY ITARATSIYA USULIDA SONLI ECHISHNING MATEMATIK INSTRUMENTI VA DASTURIY TA’MINOTI

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Annotatsiya. *Maqolada chiziqli integral tenglamalarni oddiy itatsiya usuli hamda kompyuter matematikasi tizimi vositasida sonli yechishning matematik-dasturiy ta’minoti keltirilgan.*

Аннотация. *В статье приведена математико- программное обеспечение для численного решения линейных интегральных уравнений численным методом простой итарации и средствами систем компьютерной математики.*

Abstract: *The article presents mathematical software for the numerical solution of linear integral equations using the numerical method of simple iteration and means of computer mathematics systems.*

Kalit so‘zlar. *Integral tenglamalar, diskretizatsiyalash, mathcad, maple, chiziqli va nochiziqli integro-differentsial, integral tenglamalar.*

Ключевые слова. *Интегральные уравнения, дискретизация, mathcad, кленовый, линейные и нелинейные интегро-дифференциальные, интегральные уравнения.*

Keywords. *Integral equations, discretization, mathcad, maple, linear and nonlinear integro-differential, integral equations.*

Metalurgiya sanoti, tog‘-kon sanoati, paxtachilik sanoati, to‘qimachilik sanoati, umuman olganda og‘ir va engil sanoat mashina-mexanizmlari, boshqacha aytganda qattiq jismlar mexanikasining plastinka va qobiq tipidagi yupqa devorli konstruksiyalarining dinamik tebranishlari harakat tenglamalari ayrim hollarda fazoviy o‘zgaruvchilar bo‘yicha diskretizatsiyalash usullaridan biri qo‘llanilgandan keyin chiziqli yoki chiziqsiz(nochiziqli) integro-differensial tenglamalar yoki ularning sistemalariga, chiziqli yoki chiziqsiz(nochiziqli) integral tenglamalarga yoki ularning sistemalariga keltirish mumkin[1].

Kompyuter matematikasi, kompyuter industriyasi va dasturlash texnologiyalarining jadal suratlar bilan rivojlanishi ta’lim-tarbiya, ilmiy-metodik va ilmiy tadqiqot ishlarini avtomatlashtirishning asosi sifatida e’tirof etilmoqda. Zamonaviy axborot texnologiyalari sohasida qo‘lga kiritilgan yutuqlarni qo‘llash natijasida ilmiy-tadqiqot, ilmiy-metodik, ilmiy-texnik, injenerlik, moliyaviy va iqtisodiy, kimyoviy, biologik masalalarni echishni avtomatlashtirish tomon yo‘naltirilgan ko‘plab dasturiy vositalar mavjuddir. Masalan: Mathematica, Maple, Matlab, Mathcad, Derive, Scientific, Workplce, Femlab, FeexPDE kabi universal dasturiy muhitlar shular jumlasidandir. Bulardan ikkitasi Mathematica, Maple[2] professional matematiklar va ilmiy-tadqiqotlar olib boruvchi mutaxassislar tomonidan keng qo‘llanilmoqda. Mathcad[3] esa injenerlik hisob-kitob ishlarining instrumenti sifatida ishlab chiqilgan bo‘lib hozirda etarlicha murakkablikka ega bo‘lgan hisob-kitoblarni bajarishda, ilmiy-tekshirish ishlarida har xil sonli algoritmlarni va analitik almashtirishni bajarishda foydalanilmoqda[4].

Chiziqli va nochiziqli integro-differensial, integral tenglamalar va ularning tizimlarini sonli echishning samarali usullaridan biri oddiy iteratsiya sonli usuli hamda kvadratura formularini qo‘llashga asoslangan sonli usul hisoblanadi.

Ushbu maqolada quyidagi chiziqli integral tenglama qaraladi:

$$u(t) + \int_0^t R(t-\tau)\, u(\tau)^3\, d\tau = F(t) \qquad (1)$$

bu erda , u(t) – izlanayotgan funksiya, F(t)-berilgan funksiya , $R(t) = \varepsilon\, t^{(\alpha-1)}\, \mathbf{e}^{(-\beta t)}$ -yadro, $\varepsilon, \alpha, \beta$ - o‘zgarmas parametrlar.

Integral tenglamalar nazariyasi [6]da yaxshi ishlab chiqilgan. Ma’lumki, integral tenglamalar aniq echimini topish xususiy hollardagina mumkin bo‘ladi.

(1) integral tenglama taqribiy echimi oddiy iteratsiya usulida izlanadi. Buning uchun [a,b] kesmada $t_i := a + (i-1)\,h \quad i := 1\,..\,n$ tugun nuqtalar olinadi va bu nuqtalarda (1) integral tenglama quyidagi ko‘rinishda yoziladi:

$$u(t_n) + \int_0^{t_n} R(t_n - \tau)\, u(\tau)^3\, d\tau = F(t_n) \qquad (2)$$

Yoki

$$u(t_n) + \varepsilon \int_0^{t_n} (t_n - \tau)^{(\alpha - 1)}\, \mathbf{e}^{(-\beta (t_n - \tau))}\, u(\tau)^3\, d\tau = F(t_n) \qquad (3)$$

(3) tenglamaga kiruvchi integral Abel tipidagi kuchsiz maxsuslikka ega, uni hisoblashda kvadratura formulalarini qo'llash mumkin emas. Shuning uchun : $t_n - \tau = z^r,\ r = \frac{1}{\alpha}$ almashtirish yordamida maxsuslikni bartaraf qilamiz.

U holda : $\int_0^{t_n} R(t_n - \tau)\, u(\tau)\, d\tau = \varepsilon \int_0^{t_n} (t_n - \tau)^{(\alpha - 1)}\, \mathbf{e}^{(-\beta (t_n - \tau))}\, u(\tau)^3\, d\tau =$

$$= \frac{\varepsilon \int_0^{t_n^{\alpha}} \mathbf{e}^{(-\beta z^r)}\, u(t_n - z^r)^3\, dz}{\alpha}$$

$$\frac{\varepsilon \int_0^{t_n^{\alpha}} \mathbf{e}^{(-\beta z^r)}\, u(t_n - z^r)^3\, dz}{\alpha} = \frac{\varepsilon \left(\sum_{i=1}^{n} A_i\, e^{\left(-\beta S_i^{\,r}\right)}\, u(t_n - S_i^{\,r})^3 \right)}{\alpha} =$$

$$= \frac{\varepsilon \left(\sum_{i=1}^{n} A_i\, e^{(-\beta t_i)}\, u(t_n - t_i)^3 \right)}{\alpha} = \frac{\varepsilon \left(\sum_{i=1}^{n} A_i\, e^{(-\beta t_i)}\, u_{n+1-i}^{\ 3} \right)}{\alpha}$$

$$S_i = (i - 1)\, H_n,\ H_n = \frac{t_n^{\alpha}}{n - 1},\ i = 1 \,..\, n\,,$$

$$A_i = \frac{1}{2} H_n,\ A_j = H_n,\ j = 1 \,..\, n - 1,\ A_n = \frac{1}{2} H_n$$

Shundan keyin, (3) integral tenglama chiziqli algebraik tenglamalar sistemasiga keltiriladi:

$$u_n + \frac{\varepsilon\left(\sum_{i=1}^{n} A_i\, e^{(-\beta t_i)}\, u_{n-i}^{3}\right)}{\alpha} = F_n \qquad (4)$$

(4) chiziqli algebraik tenglamalar sistemasini echish uchun oddiy iteratsiya usulini qo‘llaymiz, u holda echim quyidagi ko‘rinishga keladi:

$$u_{n+1} = F_n - \frac{\varepsilon\left(\sum_{i=1}^{n} A_i\, e^{(-\beta t_i)}\, u_{n+1-i}^{3}\right)}{\alpha}$$

Kompьyuter matematikasi tizimlarida quyidagi test-misol echiladi:

$\mathbf{ORIGIN} := 1$

O‘zgarmas parametrlar

$\mathbf{a} := 0 \qquad \mathbf{b} := 1 \qquad \mathbf{N} := 10 \qquad \mathbf{h} := \frac{\mathbf{b}-\mathbf{a}}{\mathbf{N}-1}$

$\alpha := \frac{1}{4} \qquad \beta := \frac{1}{5} \qquad \varepsilon := \frac{1}{100}$

Podprogramma-funksiyalar:

YAdro

$\mathbf{R}(\mathbf{t}) := \varepsilon \cdot \mathbf{t}^{\alpha-1} \cdot \mathbf{e}^{-\beta \cdot \mathbf{t}}$ Aniq echim

$\mathbf{U}(\mathbf{t}) := \mathbf{e}^{-\beta \cdot \mathbf{t}}$ O‘ng tomon

$\mathbf{F}(\mathbf{t}) := \mathbf{U}(\mathbf{t}) + \int_0^{\mathbf{t}} \mathbf{R}(\mathbf{t}-\tau)\cdot\mathbf{U}(\tau)^3\, d\tau$

$\mathbf{H_n}$ $\mathbf{n} := 1 .. \mathbf{N}$ $\mathbf{t_n} := \mathbf{a} + \mathbf{h}\cdot(\mathbf{n}-1)$ $\mathbf{t_n} =$ $\mathbf{F}(\mathbf{t_n}) =$ $\frac{(\mathbf{t_n})^{\alpha}}{\mathbf{N}-1}$

$\frac{\mathbf{H_1}}{2}$ $\mathbf{A_n}$ $\mathbf{j} :=$ $-1)$ $:= \mathbf{H_j}$ $\mathbf{e}^{-\beta \cdot \mathbf{t}} =$ $\mathbf{u_{n+1}} :=$ ε α $\sum$ $\mathbf{i} =$ $\mathbf{i}\cdot\mathbf{e}^{-\beta}$ $+1-\mathbf{i})^3]$ $\mathbf{u} =$

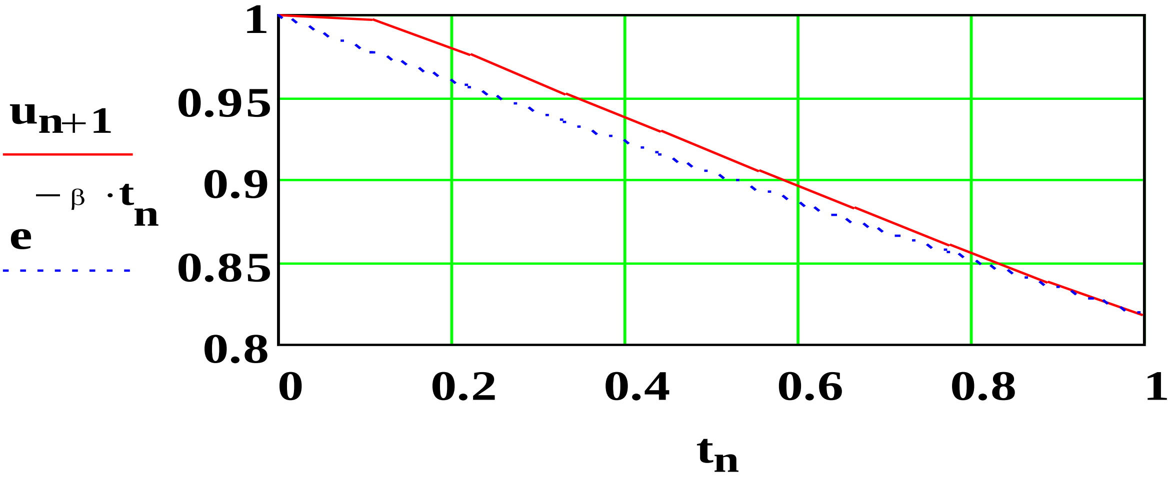
0
0.064
0.076
0.084
0.091
0.096
0.1
0.104
0.108
0.111

	1
1	1
2	0.978
3	0.957
4	0.936
5	0.915
6	0.895
7	0.875
8	0.856
9	0.837
10	0.819

0
0.111
0.222
0.333
0.444
0.556
0.667
0.778
0.889
1

1
1
0.981
0.961
0.941
0.921
0.901
0.881
0.862
0.843

	1
1	1
2	1
3	0.997
4	0.976
5	0.953
6	0.929
7	0.906
8	0.883
9	0.861
10	0.839
11	0.819



MAPLEda programma kodi:

> a:=0;b:=1;N:=10;h:=(b-a)/(N-1);

$$a := 0$$
$$b := 1$$
$$N := 10$$
$$h := \frac{1}{9}$$

> alpha:=1/4;beta:=1/5;epsilon:=1/100;

$$\alpha := \frac{1}{4}$$
$$\beta := \frac{1}{5}$$
$$\varepsilon := \frac{1}{100}$$

> R:=proc(t)epsilon*t^(alpha-1)*e^(-beta*t);end;

>

$R :=$ **proc**(t) $\varepsilon \times t$^$(\alpha - 1) \times e$^$(-\beta \times t)$ **end proc**

> U:=proc(t)exp(-beta*t);end;

$U :=$ **proc**(t) $\exp(-\beta \times t)$ **end proc**

> F:=proc(t)U(t)+Int(R(t-tau)*U(tau)^3,tau=0..t);end;

$F :=$ **proc**(t) $\mathrm{U}(t) + \mathrm{Int}(\mathrm{R}(t-\tau) \times \mathrm{U}(\tau)$^$3, \tau = 0 .. t)$ **end proc**

> for n from 1 to N do t[n]:=a+h*(n-1):

> F[n]:=exp(-beta*t[n])*(1-(epsilon/alpha)*t[n]^alpha):

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> H[n]:=t[n]^alpha/(N-1):A[1]:=H[1]/2:A[n]:=H[n]/2:
> for j from 2 to (N-1) do A[j]:=H[j]:end do:
> u[1]:=1:
> u[n+1]:=F[n]-epsilon/alpha*sum(A[i]*exp(-beta*t[i])*u[n+1-i]^3,i=1..n);
> end do:
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Xulosa

Shuni ta’kidlash mumkinki ilmiy-tatqiqot ishlarida kompьyuter matematikasi tizimlarining va maqolada keltirilgan algoritm hamda dasturning qo‘llanishi ko‘pgina muammolarning hal etilishida katta yordam beradi.

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