KOSHI MASALASINI YECHISHGA MO‘LJALLANGAN SONLI ALGORITMLARNING DASTURIY REALIZATSIYASI VA ULARNING HISOBLASH JARAYONINI TASHKIL ETISH

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Annotatsiya
Mazkur maqolada oddiy differensial tenglamalar uchun qo‘yilgan Koshi masalasini sonli yechish usullari tahlil qilinadi. Eyler, takomillashtirilgan Eyler va Runge–Kutta usullarining matematik asoslari keltiriladi hamda ularning dasturiy realizatsiyasi ko‘rib chiqiladi. Hisoblash jarayonini tashkil etishning asosiy bosqichlari, algoritmlarning aniqligi va hisoblash samaradorligi tahlil qilinadi. Sonli tajribalar natijasida usullar o‘rtasidagi aniqlik va tezkorlik jihatlari solishtiriladi.

Kalit so‘zlar: *Koshi masalasi, differensial tenglama, sonli usullar, Eyler usuli, Runge-Kutta usuli, algoritm, dasturiy realizatsiya, hisoblash jarayoni*.

SOFTWARE IMPLEMENTATION OF NUMERICAL METHODS FOR SOLVING THE CAUCHY PROBLEM AND ORGANIZING THE COMPUTATIONAL PROCESS

Abstract
This article examines numerical algorithms designed for solving the Cauchy problem for ordinary differential equations and their software implementation. The mathematical foundations of the Euler, Improved Euler, and fourth-order Runge–Kutta methods are presented and analyzed. Particular attention is given to the organization of the computational process, including the selection of step size, algorithm implementation, and result visualization. The accuracy, stability, and computational efficiency of the considered methods are compared through numerical experiments. The obtained results demonstrate that modern numerical algorithms and their software realization provide effective tools for solving differential equations arising in scientific and engineering applications.

Keywords: *Cauchy problem, ordinary differential equation, numerical methods, Euler method, Improved Euler method, Runge–Kutta method, software implementation, computational process, numerical analysis, mathematical modeling.*

Kirish: Fan va texnikaning ko‘plab masalalari differensial tenglamalar yordamida modellashtiriladi. Fizika, biologiya, iqtisodiyot, texnika va boshqa ko‘plab sohalarda jarayonlarning matematik modellarini tuzishda differensial tenglamalardan foydalaniladi. Amaliy masalalarning aksariyatida analitik yechimni topish murakkab yoki umuman mumkin emas. Shu sababli sonli usullar yordamida taqribiy yechimlarni aniqlash muhim ahamiyat kasb etadi.

Koshi masalasi oddiy differensial tenglamalar nazariyasining asosiy masalalaridan biri bo‘lib, quyidagi ko‘rinishda ifodalanadi:

$$\frac{dy}{dx} = f(x,y), \qquad y(x_0) = y_0$$

Bu yerda boshlang‘ich nuqtadagi qiymat ma’lum bo‘lib, berilgan oraliqda yechimni aniqlash talab qilinadi.

Koshi masalasini yechishning sonli usullari

Eyler usuli

Eyler usuli eng sodda sonli usullardan biri hisoblanadi. Ushbu usulning hisoblash formulasi:

$$y_{n+1} = y_n + hf(x_n, y_n)$$

Bu yerda h — integrallash qadami.

Eyler usuli hisoblash jihatidan sodda bo‘lsa-da, aniqligi nisbatan past bo‘lib, xatoligi O(h) tartibga ega.

Takomillashtirilgan Eyler usuli

Ushbu usul oraliq nuqtadagi hosila qiymatidan foydalanadi:

$$k_1 = f(x_n, y_n),$$
$$k_2 = f(x_n + h, y_n + hk_1),$$

Mazkur usul Eyler usuliga nisbatan ancha yuqori aniqlik beradi.

To‘rtinchi tartibli Runge–Kutta usuli

Amaliy hisoblashlarda eng ko‘p qo‘llanadigan usullardan biri hisoblanadi.

$$k_1 = f(x_n, y_n),$$
$$k_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right),$$

$k_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right),$

$k_4 = f(x_n + h, y_n + hk_3).$

Hisoblash formulasi:

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4$$

Ushbu usulning aniqlik tartibi $O(h^4)$ ga teng.

Algoritmlarning dasturiy realizatsiyasi

Koshi masalasini kompyuterda yechish quyidagi bosqichlardan iborat:

1. Boshlang'ich ma'lumotlarni kiritish.
2. Integrallash oralig'ini belgilash.
3. Qadam uzunligini tanlash.
4. Sonli usul formulasini qo'llash.
5. Natijalarni jadval yoki grafik ko'rinishida chiqarish.

Python dasturlash tilida Eyler usulining realizatsiyasi:

```python
def euler(f, x0, y0, h, n):
    x = x0
    y = y0

    for i in range(n):
        y = y + h * f(x, y)
        x = x + h

    return y

def f(x, y):
    return x + y

natija = euler(f, 0, 1, 0.1, 10)
print(natija)
```

Hisoblash tajribasi

Quyidagi Koshi masalasi qaraladi:

$$\frac{dy}{dx} = x + y, \quad y(0) = 1$$

Analitik yechim:

$$y = 2e^x - x - 1.$$

Turli usullar yordamida olingan natijalar analitik yechim bilan taqqoslanadi. Tajribalar shuni ko‘rsatadiki, Runge–Kutta usuli eng yuqori aniqlikni ta’minlaydi, biroq hisoblash amallari soni ko‘proq bo‘ladi.

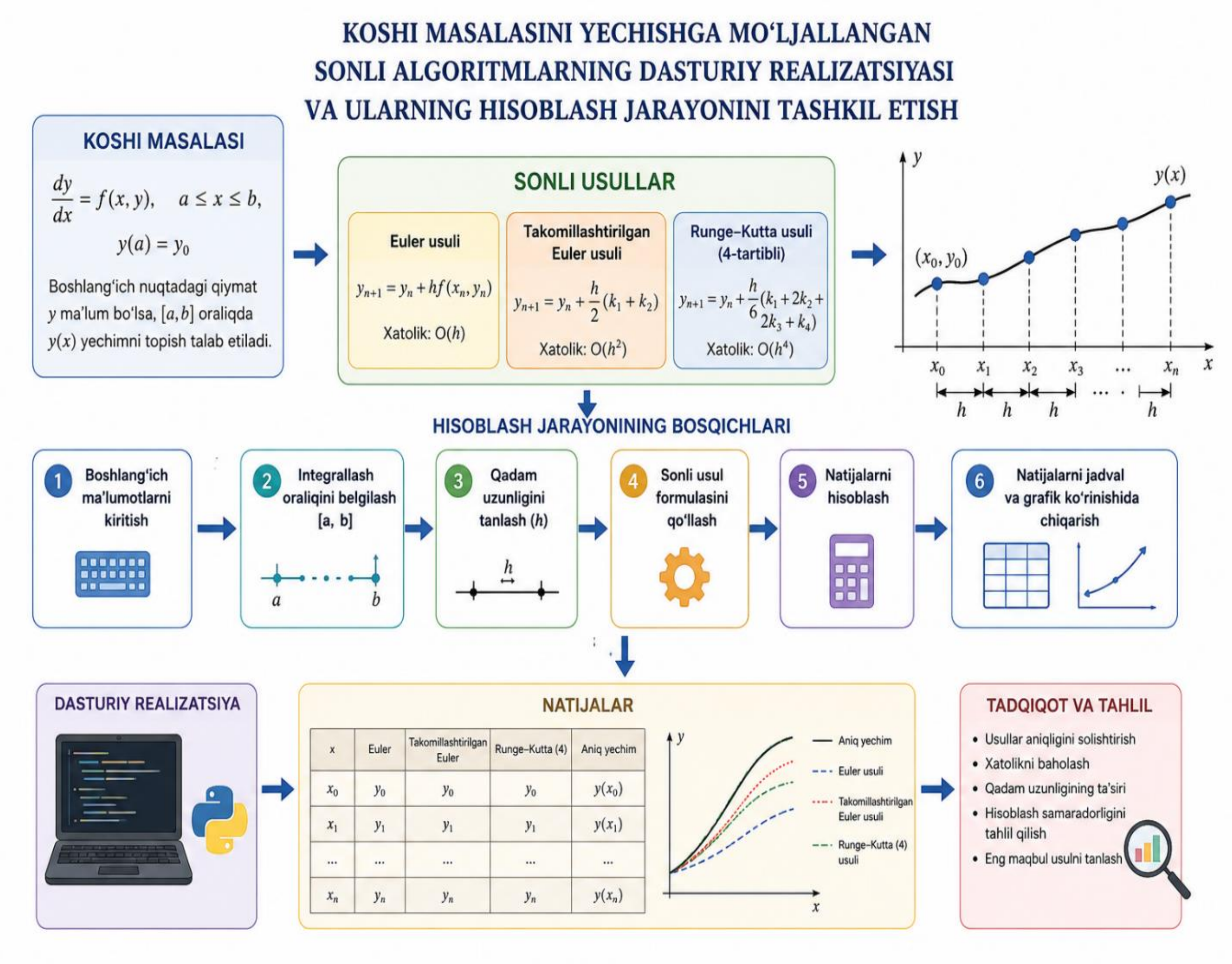
Natijalar tahlili

Sonli tajribalar natijasida quyidagi xulosalarga kelish mumkin:

- Eyler usuli sodda va tez ishlaydi.
- Takomillashtirilgan Eyler usuli aniqlikni sezilarli oshiradi.
- Runge–Kutta usuli yuqori aniqlikka ega.
- Qadam uzunligi kamayishi bilan xatolik kamayadi.
- Dasturiy realizatsiya hisoblash jarayonini avtomatlashtirish imkonini beradi.

Xulosa

Koshi masalasini sonli yechish usullari zamonaviy ilmiy va muhandislik hisoblashlarida muhim o‘rin egallaydi. Eyler, takomillashtirilgan Eyler va Runge–Kutta usullari yordamida differensial tenglamalarning taqribiy yechimlarini yuqori aniqlik bilan topish mumkin.



Dasturiy realizatsiya esa hisoblash jarayonining samaradorligini oshiradi va katta hajmdagi masalalarni qisqa vaqt ichida yechish imkonini yaratadi.

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