SONNING BUTUN QISMI VA *E* SONI BILAN BOG'LIQ MASALALAR

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1 –ta'rif. x dan oshmaydigan eng katta butun songa x ning butun qismi deyiladi va $[x]$ kabi belgilanadi.

Ta'rifga ko'ra $[x]\le x$, tengsizlik o'rinlidir. Ikkinchi tomondan $[x]+1>x$ tengsizlik ham o'rinlidir. Shunday qilib, x ning butun qismi, $[x]$ ushbu $[x]\le x<[x]+1$ qo'shtengsizlikni qanoatlantiruvchi butun sondir.

Masalan quyidagi,

$$3<\pi<4\ ,\quad 5<\frac{17}{3}<6\ ,\quad -2<\left(-\sqrt{2}\right)<(-1)\ ,\quad 5=5<6$$

munosabatlarga ko'ra,

$$[\pi]=3\ ,\quad \left[\frac{17}{5}\right]=5\ ,\quad [-2]=-2\ ,\quad [5]=5$$

tengliklar o'rinlidir. Miqdorning butun qismini, topishni bilish taqribiy hisoblashlarda muhim ahamiyatga egadir.

Sonning butun qismini topish bilan bog'liq ba'zi masalalarni qaraymiz.

1 –misol. $x=1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{5}}$ sonning butun qismini toping.

Yechish: kvadratik ildizni 0.1 aniqlikda taqribiy hisoblab,

$$1\le 1\le 1\ ,\qquad 0{,}7<\sqrt{\frac{1}{2}}<0{,}8\ ,\quad 0{,}5<\sqrt{\frac{1}{3}}<0{,}6\ ,$$

$$0{,}5\le\sqrt{\frac{1}{4}}\le 0{,}5\ ,\qquad 0{,}4<\sqrt{\frac{1}{5}}<0{,}5$$

qo'shtengsizliklarni hosil qilamiz. Hosil bo'lgan qo'shtengsizliklarni hadma –had qo'shib, $1+0.7+0.5+0.5+0.4<x<1+0.8+0.6+0.5+0.5$ ni ya'ni, $3.1<\mathrm{x}<3.4$ ni hosil qilamiz. Demak, $[x]$=3 ekan.

2 –misol. Ushbu $y=1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+.....+\frac{1}{\sqrt{1000000}}$ sonning butun qismini toping.

Yechish: bu masala avvalgi masaladan faqat qo'shiluvchilarning soni bilan farq qilmoqda, II.3.1–misolda beshta qo'shiluvchi, II.3.2–misolda 1000000 ta qo'shiluvchi ishtirok qilmoqda. Lekin, II.3.2–misolni, II.3.1–misolda qo'llanilgan usul bilan yechib bo'lmaydi.

Masalani yechish maqsadida,

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+....+\frac{1}{\sqrt{n}}$$

yig'indini o'rganamiz. Shu maqsadda

$$2\sqrt{n+1}-2\sqrt{n}<\frac{1}{\sqrt{n}}<2\sqrt{n}-2\sqrt{n-1} \qquad \text{(II.3.1)}$$

tengsizlikni isbotlaymiz. Bizga ma'lumki,

$$2\sqrt{n+1}-2\sqrt{n}=\frac{2\left(\sqrt{n+1}-\sqrt{n}\right)\left(\sqrt{n+1}+\sqrt{n}\right)}{\sqrt{n+1}+\sqrt{n}}=\frac{2}{\sqrt{n+1}+\sqrt{n}}$$

va $\sqrt{n+1}>\sqrt{n}$, shu sababli, $2\sqrt{n+1}-2\sqrt{n}<\frac{2}{2\sqrt{n}}=\frac{1}{\sqrt{n}}$. Bu esa (II.3.1) tengsizlik chap tomonining isbotini bildiradi. (II.3.1) tengsizlikning o'ng tomoni ham shunga o'xshab isbotlanadi. (II.3.1) tengsizliklarda $n=2,3,4,.....,n$ deb olib,

$$2\sqrt{3}-2\sqrt{2}<\frac{1}{\sqrt{2}}<2\sqrt{2}-2$$

$$2\sqrt{4}-2\sqrt{3}<\frac{1}{\sqrt{3}}<2\sqrt{3}-2\sqrt{2}$$

$$2\sqrt{5}-2\sqrt{4}<\frac{1}{\sqrt{4}}<2\sqrt{4}-2\sqrt{3}$$

. . . .

$$2\sqrt{n+1}-2\sqrt{n}<\frac{1}{\sqrt{n}}<2\sqrt{n}-2\sqrt{n-1}$$

qo'shtengsizliklarni hosil qilamiz. Ularni hadma –had qo'shib,

$$2\sqrt{n+1}-2\sqrt{2}<\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+.....+\frac{1}{\sqrt{n}}<2\sqrt{n}-2$$

ga ega bo'lamiz. Hosil bo'lgan tengsizliklarning har bir qismiga 1 ni qo'shib,

$$2\sqrt{n+1}-2\sqrt{2}+1<1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+.....+\frac{1}{\sqrt{n}}<2\sqrt{n}-1 \qquad \text{(II.3.2)}$$

ni hosil qilamiz. $2\sqrt{2}<3$ va $\sqrt{n+1}>\sqrt{n}$ tengsizliklarga ko'ra, (II.3.2) qo'shtengsizlikdan

$$2\sqrt{n}-2<1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+.....+\frac{1}{\sqrt{n}}<2\sqrt{n}-1 \qquad \text{(II.3.3)}$$

qo'shtengsizlikni hosil qilamiz. (II.3.3) tengsizlikdan $y=1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+.....+\frac{1}{\sqrt{1000000}}$ sonning butun qismini oson hisoblash mumkin. Buning uchun (II.3.3) qo'shtengsizlikda $n=1000000$ deb olib,

$$2\sqrt{1000000}-2<1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+.....+\frac{1}{\sqrt{1000000}}<2\sqrt{1000000}-1$$

yoki, $1998<y<1999$ ni hosil qilamiz. Demak, $[y]=1998$, ekan. **3 –misol.** $x=\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot.....\cdot\frac{99}{100}<\frac{1}{10}$ tengsizlikni isbotlang.

Yechish: quyidagicha belgilash olamiz $y=\frac{2}{3}\cdot\frac{4}{5}\cdot\frac{6}{7}\cdot.....\cdot\frac{100}{101}$. Ushbu

$\frac{1}{2}<\frac{2}{3}$, $\frac{3}{4}<\frac{4}{5}$, $\frac{5}{6}<\frac{6}{7}$, ... , $\frac{99}{100}<\frac{100}{101}$,

tengsizliklarga ko'ra, $x<y$ ekanligini va shu sababli

$$x^2<xy=\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdot\frac{5}{6}\cdot\frac{6}{7}\cdot.....\frac{99}{100}\cdot\frac{100}{101}=\frac{1}{101}$$

bo'lishini hosil qilamiz. Tengsizlikning har ikkala tomonidan kvadratik ildiz chiqarib, $x<\frac{1}{\sqrt{101}}<0.1$ ni hosil qilamiz.

***e* soni.**

Ma'lumki, e soni matemtikada muhim o'rin egallaydi. Bu sonning ta'rifini II.1.2-teorema yordamida yechiladigan bir nechta masaladan keyin keltiramiz.

4 –misol. Istalgan musbat $a,b(a\neq b)$ sonlar uchun $\sqrt[n+1]{ab^n}<\frac{a+nb}{n+1}$ tengsizlikni isbotlang.

Yechish: quyidagi $\sqrt[n+1]{ab^n}=\sqrt[n+1]{a\underbrace{bb......b}_{n}}<\frac{a+\overbrace{b+b+....+b}^{n}}{n+1}=\frac{a+nb}{n+1}$ munosabatlar II.3.4-misol isbotini yakunlaydi.

5 –misol. n soni ortib borishi bilan $x_n=\left(1+\frac{1}{n}\right)^n$ va $z_n=\left(1-\frac{1}{n}\right)^n$ miqdorlar ham o'sib borishini, ya'ni $x_n<x_{n+1}=\left(1+\frac{1}{n+1}\right)^{n+1}$, $z_n<z_{n+1}=\left(1-\frac{1}{n+1}\right)^{n+1}$ ekanligini isbotlang.

Yechish: II.1.5-misolda isbotlangan tengsizlikda $a=1$, $b=1+\frac{1}{n}$ deb olib,

$$\sqrt[n+1]{1\cdot\left(1+\frac{1}{n}\right)^n}<\frac{1+n(1+\frac{1}{n})}{n+1}=\frac{n+2}{n+1}=1+\frac{1}{n+1}$$

ni olamiz. Oxirgi tengsizlikning har ikkala tomonini $n+1$ - darajaga ko'tarib,

$\left(1+\frac{1}{n}\right)^n<\left(1+\frac{1}{n+1}\right)^{n+1}$, ya'ni $x_n<x_{n+1}$ ni hosil qilamiz. Ikkinchi tengsizlik ham xuddi shunday isbotlanadi.

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