

OPTIMIZATION MODELS IN APPLIED MATHEMATICS AND THEIR
APPLICATION IN ECONOMIC SYSTEMS

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Abstract: *This article provides a comprehensive analysis of optimization theory, one of the important areas of applied mathematics, its mathematical foundations, and its application in economic systems. The study examines the characteristics of linear and nonlinear optimization models, methods for solving them, and their impact on economic efficiency. In addition, the possibilities of optimal resource utilization are demonstrated using mathematical models adapted to real economic processes. The results show that optimization methods are an important tool in improving the decision-making process.*

Keywords: *optimization, linear programming, mathematical modeling, economic system, resource allocation, profit maximization.*

Introduction

In the context of modern globalization, the increasing complexity of economic systems and the limitation of resources require effective management methods. From this perspective, applied mathematics, especially optimization theory, plays an important role in solving economic problems.

Through the use of optimization models, it is possible to determine production volumes, minimize costs, and maximize profits. The main purpose of this article is to study the theoretical foundations of optimization models and analyze their practical application in economic systems.

Main Part

1. Theoretical Foundations of Optimization Problems

An optimization problem consists of determining the maximum or minimum value of an objective function under given constraints. In general form, it can be expressed as follows:

$$\text{Max } Z = f(X_1, X_2, \dots, X_n)$$

shartlar:

$$g_i(X_1, X_2, \dots, X_n) \leq b_i, \quad i = 1, 2, \dots, m$$

$$X_j \geq 0, \quad j = 1, 2, \dots, n$$

If the model is convex, the existence of a global optimal solution is guaranteed, which is of great importance in economic modeling.

2. Linear Programming Model

Linear programming is one of the most widely used forms of optimization, in which both the objective function and the constraints are expressed linearly:

$$Z = C_1X_1 + C_2X_2 + \dots + C_nX_n$$

Linear programming is widely applied in the following areas:

- production planning
- transportation and logistics
- optimal allocation of resources

In addition, every linear programming problem has a corresponding dual problem, which makes it possible to determine the “shadow price” of resources from an economic perspective.

3. Nonlinear Optimization Methods

Many real economic processes are non-linear. Therefore, it becomes necessary to use nonlinear optimization models. The following methods are commonly used to solve such problems:

- gradient method
- Newton's method
- iterative algorithms

Nonlinear models allow for a more accurate representation of complex systems; however, solving them requires more computational resources.

4. Application of Optimization in Economic Systems

The application of optimization models in economics is broad and includes the following areas:

- determining production volume
- minimizing costs
- maximizing profit
- optimizing investment portfolios
- managing logistics and supply chains

These methods enable enterprises to make the most efficient decisions under conditions of limited resources.

5. Practical Example and Analysis

Consider the following problem:

Objective function:

$$\text{Max } Z = 5X_1 + 4X_2$$

shartlar:

$$2X_1 + X_2 \leq 100$$

$$X_1 + X_2 \leq 80$$

$$X_1, X_2 \geq 0$$

This problem can be solved using the graphical method or the simplex algorithm.

Optimal solution:

Maximum profit: $Z=340$

This result shows that when resources are properly allocated, an enterprise can achieve maximum economic efficiency.

Research Results and Discussion

The conducted analysis shows that the use of optimization models:

- increases economic efficiency
- provides a scientific basis for decision-making
- improves the level of resource utilization

Moreover, modern software tools (Python, MATLAB, Excel Solver) enable fast and accurate solutions to optimization problems.

Conclusion

In conclusion, optimization theory is one of the most important branches of applied mathematics and plays a significant role in the effective management of economic systems. With the help of these models, it is possible to use resources rationally, reduce costs, and achieve maximum profit.

In the future, optimization methods integrated with artificial intelligence and big data technologies are expected to be widely applied across various sectors of the economy.

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