HAR XIL CHEKSIZLIKDA TURLI N - ZONALI POTENSIALLARGA YAQIN POTENSIALLI SHTURM - LIUVILL OPERATORI UCHUN ALMASHTIRISH OPERATORLARI

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Annotatsiya: *Ushbu ishda Shturm-Liuvill operatori uchun Yosta yechimlarining mavjudligi ko'rsatilgan va almashtirish operatori qurilgan.*

Kalit so'zlari: *Shturm-Liuvill operatori, xos qiymat, xos funksiya, spektr, chekl izonali potensial, almashtirish operatori, integral tenglama.*

Annotation: *This work shows the existence of Yosta solutions for the Shturm-Liuvill operator, and the substitution operator is constructed.*

Keywords: *Sturm-Liouville operator, eigenvalue, eigenfunction, spectr, finite-zone potential, substitution operator, integral equation.*

Kirish

Shturm-Liuvill operatorlari spektral nazariyasining teskari masalasini o'rganishda almashtirish operatorlari muhim rol o'ynaydi. Ular ikkita har xil Shturm-Liuvill tenglamalarining yechimlarini o'zaro bog'laydi. Almashtirish operatorlari ilk bor B.M. Levitan va J. Delsartlarning ilmiy ishlarida keltirilgan. Bu operator ixtiyoriy Shturm-Liuvill tenglamasi uchun A.Povzner tomonidan qurilgan. Spektral analizning teskari masalasini yechishda almashtirish operatorlari I.M.Gelfand, B.M.Levitan va V.A.Marchenkolar tomonidan qo'llanilgan.

Teskari masalani yechishning bir nechta usullari bor. Bu usullar ichida Gelfand-Levitan usuli muhim o'rinni egallaydi. Bu usulda almashtirish operatori asosiy rolni o'ynaydi. Usulning asosiy bosqichlaridan biri, almashtirish operatorlarining yadrosiga nisbatan olingan chiziqli integral tenglamadir. Bu integral tenglama teskari masalaning asosiy integral tenglamasi yoki Gelfand-Levitan integral tenglamasi deb yuritiladi. Ushbu ishda har xil cheksizlikda turli N - zonali potensiallarga yaqin potensialli Shturm-Liuvill operatori uchun Yosta yechimlarining mavjudligi ko'rsatilgan va almashtirish operatorlari qurilgan.

Asosiy qism

$L_2(R^1)$ fazoda quyidagi differensial ifoda bilan hosil qilingan o'z-o'ziga qo'shma operatorni qaraymiz

$$Hy \equiv -y'' + q(x)y = \lambda y, \quad (-\infty < x < \infty) \tag{1}$$

bu yerda $q(x)$ funksiya haqiqiy bo'lib,

$$(1+x^2)\left|q(x) - q_N^{(1)}(x)\right| \in L_1(0, \infty), \quad (1+x^2)\left|q(x) - q_N^{(2)}(x)\right| \in L_1(-\infty, 0). \tag{2}$$

shartlarni qanoatlantiradi, $q_N^{(1)}(x)$, $q_N^{(2)}(x)$ funksiyalar umumiy holda davriy bo'lmagan turli chekli zonali potensiyallar.

Ushbu maqolada (2) potensiallar sinfida (1) operator uchun Yosta yechimlarining mavjudligi ko'rsatilgan va almashtirish operatori qurilgan.

Spektrlari ustma-ust tushuvchi o'z-o'ziga qo'shma

$$H_N^{(j)} y \equiv -y'' + q_N^{(j)}(x)y = \lambda y, \; (-\infty < x < \infty), \; j = 1, 2, \tag{3}$$

operatorlarni qaraymiz,

$$\sigma(H_N^{(1)}) = \sigma(H_N^{(2)}) = E_N = [0, \lambda_1] \cup [\mu_1, \lambda_2] \cup \ldots \cup [\mu_N, \infty).$$

(3) tenglamalarning Veyl-Blox yechimlari [3]

$$\varphi_{1,2}(x, \lambda) = \sqrt{\frac{P(x,\lambda)}{P(\lambda)}} \exp\left\{\pm i\sqrt{R(\lambda)} \int_0^x \frac{du}{P(u,\lambda)}\right\}, \quad j = 1, \text{ bo'lsa}$$

$$\psi_{1,2}(x, \lambda) = \sqrt{\frac{Q(x,\lambda)}{Q(\lambda)}} \exp\left\{\pm i\sqrt{R(\lambda)} \int_0^x \frac{du}{Q(u,\lambda)}\right\}, \quad j = 2, \text{ bo'lsa}$$

ko'rinishda bo'ladi, bu yerda

$$R(\lambda) = \lambda \prod_{k=1}^{N} (\lambda - \lambda_k)(\lambda - \mu_k), \; P(x, \lambda) = \prod_{k=1}^{N} \left(\lambda - \xi_k^{(1)}(x)\right),$$

$$Q(x, \lambda) = \prod_{k=1}^{N} \left(\lambda - \xi_k^{(2)}(x)\right),$$

$\xi_k^{(j)}(t)$ $k = 1, 2, \ldots, N$, $j = 1, 2$ lar $q_N^{(j)}(x+t)$ potensiallarga mos keluvchi spektral parametrlar.

$\overset{0}{\Lambda}$ orqali $H_N^{(j)}$, $j = 1,2$ operatorning spektral sirtini belgilaymiz ([4]). Bu sirt $H_N^{(j)}$, $j = 1,2$ operatorning $E_N = [0, \lambda_1] \cup [\mu_1, \lambda_2] \cup \ldots \cup [\mu_N, \infty)$ spektr zonalari bo'yicha qirqimidagi ikkita $C \setminus E_N$, spektral tekislikni yopishtirishdan hosil boladi.

$f(x, \lambda)$, $g(x, \lambda)$ orqali (1) tengalamaning

$$\lim_{x \to \infty} \left(f(x, \lambda) - \varphi_1(x, \lambda)\right) = 0, \quad \lim_{x \to -\infty} \left(g(x, \lambda) - \psi_2(x, \lambda)\right) = 0, \; \lambda \in \overset{o}{E}_N$$

shartlarni qanoatlantiruvchi yechimlarini belgilaymiz.

1-Teorema. 1) $\lambda \in \overset{o}{E}_N$ bo‘lganda (1) tenglama quyidagi ko‘rinishda ifodalanuvchi chziqli bog‘lanmagan Yosta yechimlarga ega bo‘ladi:

$$f(x,\lambda)=\varphi_1(x,\lambda)+\int_x^\infty K^+(x,t)\varphi_1(t,\lambda)\,dt \equiv (\mathrm{I}+\mathrm{K}^+)\,\varphi_1(x,\lambda),$$

$$\overline{f(x,\lambda)}=\varphi_2(x,\lambda)+\int_x^\infty K^+(x,t)\varphi_2(t,\lambda)\,dt \equiv (\mathrm{I}+\mathrm{K}^+)\,\varphi_2(x,\lambda), \qquad (4)$$

bu yerda $K^+(x,t)$ (almashtirish operatorining yadrosi) quyidagi integral tenglamaning yechimi bo‘ladi

$$K^+(x,t)=\int_{\frac{x+t}{2}}^\infty \Gamma^{(1)}(x,s,s,t)[q(s)-q_N^{(1)}(s)]ds+$$

$$+2\int_{\frac{x+t}{2}}^\infty d\alpha\int_0^{\frac{t-x}{2}}[q(\alpha-\beta)-q_N^{(1)}(\alpha-\beta)]\,\Gamma^{(1)}(x,\alpha-\beta,\alpha+\beta,t)K^+(\alpha-\beta,\alpha+\beta,)d\beta\,,$$

bu yerda

$$\Gamma^{(1)}(x,t,y,z)=\frac{1}{2\pi}\int_{E'_N} F^{(1)}(x,t;s^2)\,\varphi_1(y,s^2)\varphi_2(z,s^2)\,\frac{P(s^2)}{\sqrt{R(s^2)}}\,sds, \quad (t\ge x),$$

bundan tashqari $K^+(x,t)=0, \quad x>t.$

2) $\lambda \in \overset{o}{E}_N$ bo‘lganda (1) tenglama quyidagi ko‘rinishda ifodalanuvchi chziqli bog‘lanmagan Yosta yechimlarga ega bo‘ladi:

$$g(x,\lambda)=\psi_2(x,\lambda)+\int_{-\infty}^x K^-(x,t)\psi_2(t,\lambda)dt \equiv (\mathrm{I}+\mathrm{K}^-)\psi_2(x,\lambda),$$

$$\overline{g(x,\lambda)}=\psi_1(x,\lambda)+\int_{-\infty}^x K^-(x,t)\psi_1(t,\lambda)dt \equiv (\mathrm{I}+\mathrm{K}^-)\psi_1(x,\lambda), \qquad (5)$$

bu yerda $K^-(x,t)$ (almashtirish operatorining yadrosi) quyidagi integral tenglamaning yechimi bo‘ladi

$$K^-(x,t)=\int_{-\infty}^{\frac{x+t}{2}} \Gamma^{(2)}(x,s,s,t)[q(s)-q_N^{(2)}(s)]ds+$$

$$+2\int_{-\infty}^{\frac{x+t}{2}} d\alpha\int_0^{\frac{x-t}{2}}[q(\alpha+\beta)-q_N^{(2)}(\alpha+\beta)]\,\Gamma^{(2)}(x,\alpha+\beta,\alpha-\beta,t)K^-(\alpha+\beta,\alpha-\beta)d\beta$$

,

bu yerda

$$\Gamma^{(2)}(x,t,y,z)=\frac{1}{2\pi}\int_{E'_N}F^{(2)}(x,t;s^2)\psi_1(y,s^2)\psi_2(z,s^2)\frac{Q(s^2)}{\sqrt{R(s^2)}}sds,\quad (t\le x),$$

bundan tashqari $K^{-}(x,t)=0,\ t>x.$

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