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TRIGONOMETRIYANI VUJUDGA KELISHI, RIVOJLANISHI VA TRIGONOMETRIK IFODALARNI HISOBLASH

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Annotatsiya. *Mazkur maqola matematikaning trigonometriya bo‘limiga oid bo‘lib, trigonometriyaning kelib chiqish tarixi, vujudga kelishi, geometriya va astranomiya fani bilan bog‘liqligi aytib o‘tilgan. Shuningdek O‘rta Osiyo olimlari trigonometriya rivojiga qo‘shgan hissalari hamda uning amaldaga tatbiqlari haqida ochib berilgan va trigonometrik ifodaning qiymatlarini hisoblashda hamda trigonometrik tenglamani yechishda ba’zi trigonometrik formulalardan foydalanib yechilgan masalalar.*

Kalit so‘zlar: *Trigonometriya, astronomiya, uchburchak, sinus, cosinus, tangens, kotangens, vatar, gipotenuza.*

Аннотация. *Эта статья посвящена тригонометрическому разделу математики, и упоминается, что тригонометрия связана с историей возникновения, создания, геометрии и астрономии. Также раскрывается вклад ученых Центральной Азии в развитие тригонометрии и ее практическое применение в вычислении значений тригонометрических выражении задач с использованием некаторых тригонометрических формул при решении тригонометрических уравнений.*

Ключевые слова: *Тригонометрия, астрономия, триуголник, синус, косинус,тангенс, котангенс, ватар, гепотенуза.*

Annotation. *This article is about the trigonometry branch of mathematics , it is mentioned that the history of the origin of trigonometry, its creation, and its connection with the science of geometry and astronomy. Also, the contributions of Central Asian scientists to the development of trigonometry and its practical applications were revealed and problems solved using some trigonometric formulas in calculating the value of trigonometric expressions and solving trigonometric equations.*

Key words: *Trigonometry, astronomy, triangle, sine, cosine, tangent, cotangent, watar, hypotenuse.*

Kirish. Trigonometriya (yunonchadan “trigon”- uchburchak, “metrezis”-o‘lchash so‘zidan olingan bo‘lib “uchburchaklarni o‘lchash” deb ataladi. Trigonometriya matematikaning asosiy bo‘limlaridan biri. U uchburchak tomonlari va burchaklari orasidagi bog‘lanishlarni o‘rganadi.

Trigonometriyaning vujudga kelishi. Er. av .II asrda astronomiyada bir qancha masalalar to‘planib qoladi. Bunga matematika fani zarur edi.Trigonometriya astronomiyaning ehtiyojlaridan kelib chiqqan bo‘lib, uzoq obyektlarni o‘lchashda astronomiyada xizmat qilardi. Triigonometriya uchburchaklarni hisoblashda ishlatilganligi sababli olimlar uni geometriya tarkibiga o‘tkazishmoqchi bo‘lishadi. Lekin yunonlar bunga qarshi bo‘lishdi. Trigonometriya astronomiyada zarurligi sabab,astronomiya tarkibida qoladi.

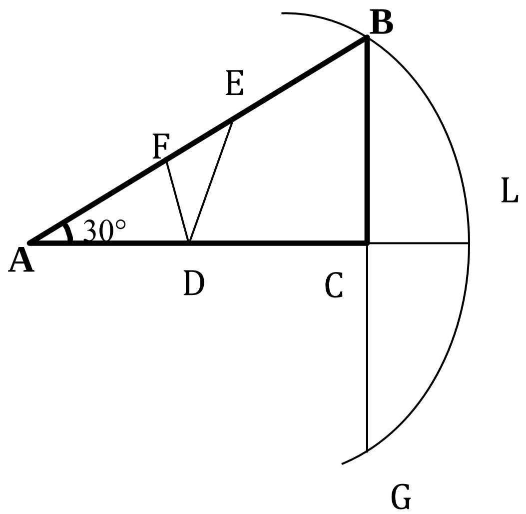
Demak,uzoqlik, to‘g‘ri chiziq, burchak biror figuralarning elementlari bo‘lishi,trigonometriya shu figuralarning orasidagi bog‘lanishni o‘rganishi kerak edi.

Masalan, biror bir uchburchak va ular orasidagi burchaklar yig‘indisi 180° ga teng. Bundan uchburchakning ikkita burchagidan uchunchi burchagini toppish mumkin. Shuningdek ixtiyoriy uchburchakni to‘g‘ri burchakli uchburchakka ajratish mumkin. Bu esa trigonometriyaning asosiy masalalari hisoblanadi.Agar ABC to‘g‘ri burchakli uchburchak berilgan bo‘lsin. A A o‘tkir burchagi AB shu uchburchakning gepotenuzasi. Demak $\angle B = 180^{\circ} - \angle A - \angle C$. Bizga $\angle A$ ma’lum bo‘lsa, $\frac{BC}{AB}$ ni qarshisidagi katetning gepotenuzasini topamiz:

$$\frac{ED}{AE} = \frac{FD}{AD} = \frac{BC}{AB}$$

Shunday qilib, katetning gepotenuzaga nisbati burchakning funksiyasi, yani funksiyaning **sinusi** deb ataladi va ***sin*** ko‘rinishda belgilanadi.

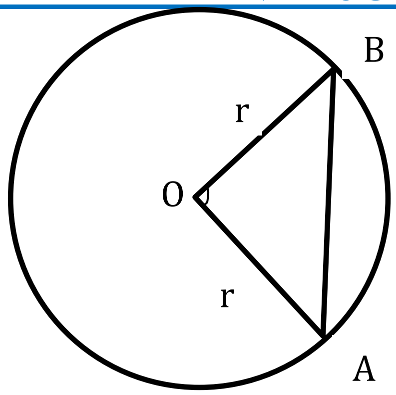
Sinusni hisoblashni geometriya yordamida topishni ko‘rib chiqamiz $\angle A = 30^{\circ}$ bo‘lsa AA ni markaz qilib, AB radusi aylana chizamiz va BC kesmani davom ettiramiz, u aylana bilan G nuqtada kesishadi $AC \perp BC$ bo‘lsa $CG = BC$, $\cup BL = \cup LG$, $\angle GAL = 30^{\circ}, BC$ vatar aylanaga ichki chizilgan C burchakning tomoni, uning radiusga nisbati $\frac{BC}{AC} = \frac{1}{2} = sin 30^{\circ}$ (1-rasm).



1-rasm.

Bu masalani er. av. II asirda Gipparx fanga kiritgan. Shu asrda fanga o‘z hissasini qo‘shgan yana bir olim Ptolemey “Almagest” asarida vatar bilan bog‘liq bo‘lgan trigonometriyani to‘liq bayon qilgan. Ptolemey aylanaga tortilgan vatarni trigonometriyaga bog‘lab topgan.

Masalan. Markaziy α burchakli r radiusli doira vatarning uzunligi $2r sin\frac{\alpha}{2}$ ga teng (2-rasm).



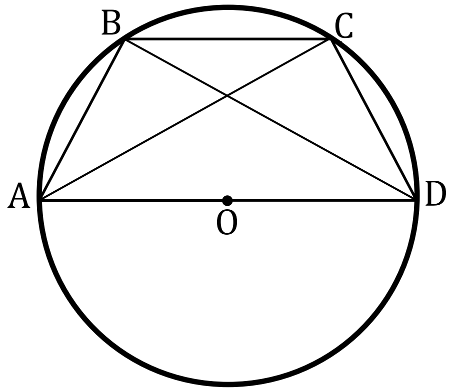
2-rasm.

Faraz qilaylik, doiraga $ABCD$ to‘g‘ri burchak ichki chizilgan bo‘lsin.

Lemma. AC va BD dioganallardan hosil bo‘lgan to‘g‘ri to‘rtburchak yuzi berilgan to‘rtburchak qarama-qarshi tomonlaridan hosil bo‘lgan to‘g‘ri to‘rtburchaklar yuzlari yig‘indisiga teng deyiladi.

$$AB \cdot BD = AB \cdot CD + AD \cdot BC$$ [2]

Bu lemma AD tomon doiraning diametridan iborat bo‘lgan ichki chizilgan $ABCD$ to‘rtburchak tadbiq etilgan (3-rasm).



3-rasm.

$$BC = \frac{AC \cdot BC + AB \cdot CD}{AD}$$

[3]

Agar $\angle AOC = 2\alpha, \angle AOB = 2\beta$ va $r = 1$ desak yuqodagi munosabat shu ko‘rinishda bo‘ladi,

$$\sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \sin\beta$$

Trigonometriyaning rivojlanishi . Trigonometriya rivoji uchun o‘z hissasini qo‘shgan allomalardan biri Muso al-Xorazmiy (783-850) birinchi bo‘lib ,sinus va teskari sinusni kiritgan Al-xorazmiy sinus va teskari sinus bo‘yicha yoy topishni shunday chiroyli va ixcham tasvirlab bergan .Bu amallarni bajarishda Xorazmiy radiusini 60 birlikka teng deb olgan , 60 birlik hozirda doira radiusi 1ga teng. Bu amalni quydagicha yozish mumkin. A yoyning teskari sinusini ***sinvers α*** $\alpha<90^{\circ}\, sinver\alpha = 60 - \sin(90^{\circ} - \alpha)$

$$\alpha>90^{\circ}\, sinversa\alpha = 60 + \sin(\alpha - 90^{\circ})$$

Agar hozirgidek doira radiusini 1ga teng desak,shunga ko‘ra quydagicha belgilanadi.

$$sinversa\alpha = 1 - cos\alpha$$

Trigonometriyaning rivojiga hissa qo‘shgan yana bir olim Abu-Rayhon Beruniy (973-1048). Beruniy “Masud Qonuni” asarida trigonometriyahaqida aniq va to‘liq bayon qilingan . U trigonometriyada vatarlarni aniqlash va doirada ichki chizilgan muntazam ko‘pburchaklarning tomonlarini topish bilan shug‘ullangan.

Ketma-ket bo‘lishdan hosil bo‘ladigan vatarlarni topish qoidasi bizning belgilashlarda quydagicha

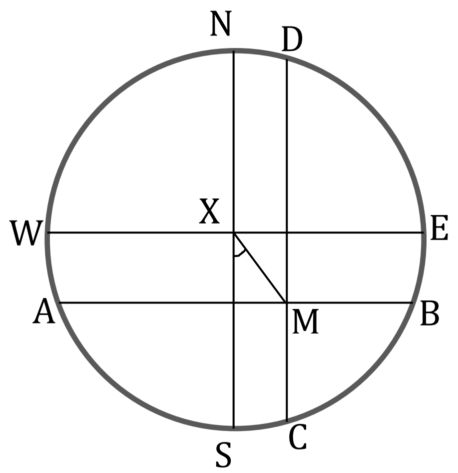
$$sin\ \frac{\alpha}{2} = \frac{1}{2}\sqrt{2\text{-}2cos\alpha}$$

$$sin\ \frac{\alpha}{2} = \frac{1}{2}\sqrt{2\text{-}\sqrt{2 + 2cos\alpha}}$$

$$sin\frac{\alpha}{2} = \frac{1}{2}\sqrt{2\text{-}\sqrt{2 + \sqrt{2\text{-}cos\alpha}}}$$

Islom dinidagi qiblani, ya’ni Makkai Mukarramadagi Ka’bani aniqlash masalasi turli trigonometrik va geografik metodlar bilan yechiigan. Bu masalaga har bir musulmon kishi bir sutkada besh marta duch keladi. Yashash hududingizda qibla qaysi tomondaligini biror belgi orqali belgilab olgansiz. Agar siz dala yoki sahroda bo‘lsangiz qanday aniqlaysiz?

Bu masala IX -XIX asrlarda keng tarqalgan soddava taqribiy metodni Abu Abdullo Muhammad ibn Battoniy (850-929) ishlab chiqqan.



4-rasm

Faraz qilaylik, markazida kuzatuvchi turgan doirada shimol-janub chizig‘i MS va g‘arb-sharq chizig‘i WE o‘tkazilgan bo‘lsin. Δφ va – Makka va kuzatuvchi joyining kengliklar ayirm, ΔL esa ular uzunliklarining farqi bo‘lsin, ya’ni EB=Δφ va SC=ΔL ga teng.

Demak, WE ga parallel AB to‘g‘ri chiziq NSga parallel DC chiziq o‘tkaziladi. Bu to‘g‘richiziq M nuqtada kesishadi , u holda Makkaga yo‘nalish $< SXM = q$ kabi aniqlanadi. M ni koordinata boshi deb qarash $x = sin\Delta L$, $y = sin\Delta\varphi$

$$q' = 90° - q = \text{arctg}\frac{\sin\Delta\varphi}{\sin\Delta L}$$

Shunday qilib

$$q = 90^{\circ} - arctg\frac{\sin\Delta\varphi}{sin\Delta L}$$

Trigonometrik masalalar. 1. Agar $ctg\alpha = \sqrt{2} - 1$ bo‘lsa, cosα ning qiymatini toping.

Trigonometrik funksiyalar orasidagi bog‘lanishlar formulasiga ko‘ra $ctg\alpha = \frac{\cos\alpha}{\sin\alpha}$ ga teng.

Bunda $\cos\alpha$ ni $\cos 2\alpha$ ga keltiramiz , buning uchun

$$1 - cos2\alpha = \sin^2 2\alpha$$
$$1 + cos2\alpha = 2\cos^2\alpha$$

formulalaridan foydalanib,

$$cos\alpha = \sqrt{\frac{1 + cos2\alpha}{2}}$$

$$sin\alpha = \sqrt{\frac{1 - cos2\alpha}{2}}$$

ko‘rinishga keltiramiz.

$$ctg\alpha = \frac{cos\alpha}{sin\alpha} = \frac{\sqrt{\frac{1 + cos2\alpha}{2}}}{\sqrt{\frac{1 - cos2\alpha}{2}}} = \frac{\sqrt{1 + cos2\alpha}}{\sqrt{1 - cos2\alpha}}$$

Demak, $ctg\alpha = \frac{\sqrt{1+cos2\alpha}}{\sqrt{1-cos2\alpha}}$ teng bo‘lsa, shartga ko‘ra tenglikdan $cos2\alpha$ ni topib olamiz

$$\frac{\sqrt{1 + cos2\alpha}}{\sqrt{1 - cos2\alpha}} = \sqrt{2} - 1$$

Tenglikning ikkala tomonini kvadratga ko‘tarib olamiz ,

$$\frac{1 + cos2\alpha}{1 - cos2\alpha} = 3 - 2\sqrt{2}$$

ko‘rinishda keltiramiz .

$$1 + cos2\alpha = (3 - 2\sqrt{2})(1 - cos2\alpha)$$
$$1 + cos2\alpha = 3 - 3cos2\alpha - 2\sqrt{2} + cos2\alpha$$
$$4cos2\alpha - 2\sqrt{2}cos2\alpha = 2 - 2\sqrt{2}$$
$$\left(4 - 2\sqrt{2}\right) \cdot cos2\alpha = 2 - 2\sqrt{2}$$

$$cos2\alpha = \frac{2-2\sqrt{2}}{4-2\sqrt{2}} = \frac{-\sqrt{2}}{2} = -\ \frac{1}{\sqrt{2}}$$

2. $sinx + ctg\frac{x}{2} = 2$ tenglamani yeching.

Tenglamani yechishda

$sinx = \frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}}$ va $ctgx = \frac{1}{tgx}$

formuladan foydalanib,

$$\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}} + \frac{1}{ctgx} = 2$$

ko‘rinishda keltirib olamiz.

So‘ng $tg\frac{x}{2} = t$ deb belgilash kiritamiz$\frac{2t}{1+t^2} + \frac{1}{t} = 2$

$$\frac{2t+1+t^2}{t\,(\,t^2+1)} = 2$$

$$t^2+2t+1 = 2\,t^3+2$$

$$2t^3 - t^2 + 1 = 0$$

$$(t-1)(2t^2-t+1)$$

Bu erda $2t^2$- t + 1≠ 0 chunki $D = b^2 - 4ac$ D= 1 - 4 · 2 = -7

Demak D<0 bo‘lsa, tenglama yechimga ega emas .

$$t-1=0$$

$$t=1$$

$$tg\frac{x}{2}=1$$

$$\frac{x}{2} = \frac{\pi}{4} + \pi k \qquad k \in Z$$

Xulosa. Xulosa qilib shuni aytish kerakki, matematikaning trigonometriya bo‘limi juda keng va katta. Demak tigonometriya birgina matematika fanida emas , balki astronomiya va geometriyada qo‘llanilgan. Tigonometriya hayotga qancha zarurligi, xizmat qilishi va hamda trigonometrik ifodalar orqali ba’zi masalalarni yechishda yana bir isbotini topdi. Trigonometrik ifodalarning qiymatlarini hisoblashda trigonometriya formulalarni bir biri bilan bog‘lanib sodda yechimlarga kelishini ajoyibligini ko‘rish mumkin.

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