

**BESH O'LCHAMLI MODEL FILIFORM 3-LI ALGEBRANING BIR  
O'LCHAMLI YECHILUVCHAN KENGAYTMASI****Beshimova Shaxnoza***Buxoro Davlat universiteti*[shaxnozabeshimova@mail.ru](mailto:shaxnozabeshimova@mail.ru)

**Annotatsiya.** *Daslab 1985-yilda Filippov  $n$ -Li algebrasi tushunchasini kiritdi va  $(n+1)$ -o'lchamli  $n$ -Li algebralarini tasnifladi.  $n$ -Li algebrasining strukturasi  $n$ -ar chiziqli operatsiya bilan bog'langan Li algebralaridan farqli.  $n=3$  holati, ya'ni ternar chiziqli operatsiya, 1-bo'lib Nambuning ishlarida uchraydi. Bu ishda Nambu Poisson qavslarini kengroq o'rgangan va ternar chiziqli qavsni  $[-,-,-]$  o'z ichiga olgan Hamilton tenglamalarini o'rgangan. Takhtajan Nambu mexanikasini umumiy algebraik va geometrik nuqtai nazardan o'rganib chiqqan va Nambu mexanikasi va  $n$ -Li algebralarining Filippov nazariyasi orasidagi bog'liqlikni isbotlagan. R.Baining ishlarida  $(n+2)$ -o'chamli  $n$ -Li algebralarining izomorfizm kriteriyalari isbotlangan va 0 xarakteristikali maydon ustidagi  $(n+1)$ -o'lchamli  $n$ -Li algebralarining hamda  $(n+2)$  o'lchamli  $n$ -Li algebralarining to'la klassifikatsiyalari berilgan.*

**Kalit so'zlar:**  *$n$ -Li algebralar, nilpotent  $n$ -Li algebralar, yechiluvchan  $n$ -algebralar, differensial.*

**Abstract.** *First, in 1985, Filippov introduced the concept of  $n$ -Lie algebras and classified  $n$ -dimensional  $n$ -Lie algebras. The structure of  $n$ -Lie algebras is different from that of Lie algebras associated with a linear operation. The case of a ternary linear operation, i.e., a 1-dimensional linear operation, is found in the works of Nambu. In this work, Nambu studied Poisson brackets more extensively and studied Hamilton equations involving ternary linear brackets. Takhtajan studied Nambu mechanics from a general algebraic and geometric point of view and proved the connection between Nambu mechanics and the Filippov theory of  $n$ -Lie algebras. In the works of R. Bai, isomorphism criteria for  $n$ -dimensional  $n$ -Lie algebras are proved and complete classifications of  $n$ -dimensional  $n$ -Lie algebras and  $(n+2)$ -dimensional  $n$ -Lie algebras over a field of characteristic 0 are given.*

**Key words:**  *$n$ -Lie algebras, nilpotent  $n$ -Lie algebras, solvable  $n$ -Lie algebras, derivation.*

**Абстракт.** *Сначала в 1985 году Филиппов ввел понятие  $n$ -алгебры Ли и классифицировал  $n$ -мерные  $n$ -алгебры Ли. Структура  $n$ -алгебр Ли отличается от структуры алгебр Ли, связанных с линейной операцией. Случай тернарной линейной операции, т. е. одномерной линейной операции, встречается в*

работам Намбу. В этой работе Намбу более подробно изучил скобки Пуассона и уравнения Гамильтона, включающие тернарные линейные скобки. Тахтаджан изучил механику Намбу с общей алгебраической и геометрической точки зрения и доказал связь между механикой Намбу и теорией Филиппова -алгебр Ли. В работах Р. Бая доказаны критерии изоморфизма  $n$ -мерных -алгебр Ли и даны полные классификации  $n$ -мерных -алгебр Ли и размерных -алгебр Ли над полем характеристики 0.

**Ключевые слова:**  $n$ -левые алгебры, нильпотентные  $n$ -левые алгебры, разрешимые  $n$ -левые алгебры, дифференцирование.

## KIRISH

**1-ta'rif.[1]**  $F$  maydon ustida aniqlangan  $L$  vektor fazoda shunday  $n$ -ar polichiziqli  $[-, -, \dots, -]$  amal mavjud bo'lib, quyidagi ayniyatlarni qanoatlantirsa:

$$[x_1, \dots, x_n] = (-1)^{\text{sign}(\sigma)} [x_{\sigma(1)}, \dots, x_{\sigma(n)}],$$

va

$$[[x_1, \dots, x_n], y_2, \dots, y_n] = \sum_{i=1}^n [x_1, \dots, [x_i, y_2, \dots, y_n], \dots, x_n]$$

u holda  $L$  algebraga  $n$ -Li algebrasi deyiladi, bu yerda  $\forall x_1, \dots, x_n, y_2, \dots, y_n \in L$  va ixtiyoriy  $\sigma \in S_n$  va  $\text{sign}(\sigma) - \sigma$  o'rinlashtirishning juft yoki toqli.

**2-ta'rif.[1]** Aytaylik,  $L$   $n$ -Li algebrasining  $D: L \rightarrow L$  chiziqli akslantirishi berilgan bo'lsin. Agar ixtiyoriy  $x_1, x_2, \dots, x_n \in L$  elementlari uchun, quyidagi tenglik o'rinli bo'lsa,

$$D([x_1, x_2, \dots, x_n]) = \sum_{i=1}^n [x_1, \dots, D(x_i), \dots, x_n],$$

u holda  $D$  chiziqli akslantirishga  $L$   $n$ -Li algebrasining differentsiallashi deyiladi.  $L$   $n$ -Li algebrasining barcha differentsiallashlari to'plami  $\text{Der}(L)$  kabi belgilanadi va u  $\text{gl}(L)$  Li algebrasining qism algebrasi bo'ladi.

Agar  $\text{ad}(x_2, x_3, \dots, x_n): L \rightarrow L$  akslantirish quyidagi tenglikni qanoatlantirsa,

$$\text{ad}(x_2, x_3, \dots, x_n)(y) = [y, x_2, x_3, \dots, x_n] \quad \forall y \in L,$$

u holda ushbu akslantirishga o'ng ko'paytma deyiladi.

Bu ko'paytma  $L$   $n$ -Li algebrasi uchun differentsiallash bo'ladi. O'ng ko'paytma operatorlarining barcha chiziqli kombinatsiyalari  $\text{Der}(L)$  Li algebrasining ideali bo'ladi va  $\text{Ad}(L)$  fazoning elementlari ichki differentsiallashlar deyiladi.

**3-ta'rif.[1]**  $L$   $n$ -Li algebrasi va  $B$  uning qism fazosi bo'lsin. Agar  $[B, B, \dots, B] \subseteq B$  munosabati o'rinli bo'lsa, u holda  $B$   $n$ -Li algebrasining qism algebrasi deyiladi.

**4-ta'rif.[1]**  $L$   $n$ -Li algebrasining  $I$  qism fazosi uchun  $[I, L, \dots, L] \subseteq I$  munosabat o'rinli bo'lsa,  $I$   $n$ -Li algebrasining ideali deyiladi.

$L n$  -Li algebrasining ixtiyoriy  $I$  ideali uchun mos ravishda quyi markaziy qator va hosilaviy qatorlarni quyidagicha aniqlaymiz:

$$I^1 = I, I^{k+1} = [I^k, I, L, \dots, L], k \geq 1,$$

$$I^{(s)} = I, I^{(s+1)} = [I^{(s)}, I^{(s)}, L, \dots, L], s \geq 1,$$

**5-ta'rif.[1]**  $L n$  -Li algebrasi bo'lsin. Agar  $\exists r \in N$  soni uchun  $I^{(r)} = 0$  munosabat o'rinli bo'lsa,  $I$  yechiluvchan ideal deyiladi, xususan agar  $\exists r \in N$  soni uchun  $L^{(r)} = 0$  tenglik o'rinli bo'lsa, u holda  $L n$  -Li algebrasi yechiluvchan  $n$  -Li algebrasi deyiladi.

**6-ta'rif.[1]** Agar  $\exists r \in N$  soni uchun  $I^r = 0$  bo'lsa, u holda  $I$  nilpotent ideal deyiladi. Agar  $\exists r \in N$  soni uchun  $L^r = 0$  tenglik o'rinli bo'lsa, u holda  $L$  nilpotent  $n$  -Li algebra deyiladi.

5 o'lchamli 3-Li algebralarining differensiallashlar fazosi

7 o'lchamli 3-Li algebraning differensiallashlar fazosini ko'rib chiqaylik:

$$N: \begin{cases} [e_3, e_1, e_2] = e_4, \\ [e_4, e_1, e_2] = e_5. \end{cases}$$

$$Der(N) = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ 0 & 0 & a_{33} & a_{34} & a_{35} \\ 0 & 0 & 0 & \delta + a_{33} & a_{34} \\ 0 & 0 & 0 & 0 & 2\delta + a_{33} \end{pmatrix}$$

bu yerda  $\delta = a_{1,1} + a_{2,2}$ .

$e'_1 = e_1, e'_2 = \alpha e_1 + e_2, e'_i = e_i, 3 \leq i \leq 5$  bazis almashtirish bajarilsa, quyidagi matritsa hosil bo'ladi:  $Der(N) = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & a_{22} & a_{23} & a_{24} & a_{25} \\ 0 & 0 & a_{33} & a_{34} & a_{35} \\ 0 & 0 & 0 & \delta + a_{33} & a_{34} \\ 0 & 0 & 0 & 0 & 2\delta + a_{33} \end{pmatrix}$

bu yerda  $\delta = a_{1,1} + a_{2,2}$ .

Model filiform olti o'lchamli yechiluvchan 3-Li algebralari tasnifi

**1-teorema.**  $R$  model filiform 6-o'lchamli yechiluvchan 3-Li algebrasi bo'lsin. U holda  $R$  da  $\{x, e_1, e_2, \dots, e_5\}$  bazis mavjud va  $R$  ning ko'paytmalari quyidagiga teng:

$$[e_3, e_1, e_2] = e_4,$$

$$[e_4, e_1, e_2] = e_5,$$

$$[x, e_1, e_4] = e_4,$$

$$[x, e_2, e_5] = 2e_5.$$

va bazis elementlarining qolgan ko'paytmalari nolga teng.

## XULOSA

Ushbu maqolada model filiform olti o'lchamli yechiluvchan 3-Li lgebralari tasnifi o'rganilgan. Kichik o'lchamli model filiform 3-Li algebralarining tasnifini olishda berilgan algebraning differensiallashlar fazosi matritsaviy ko'rinishi o'rganildi va uni bazis almashtirishlar yordamida yuqori uchburchak shaklga keltirilib, mos chiziqli erkli differensiallashlar fazosiga ichki differensiallashlarni akslantirish yordamida bir

nechta ko'paytmalarni olamiz. Olingan ko'paytmalarni Umumlashgan Yakobi ayniyatidan foydalanib ba'zi koeffitsiyentlar nolga tengligi olingan.

### FOYDALANILGAN ADABIYOTLAR

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